Narrow Bandstop Filters

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Abstract—**The synthesis of narrow bandstop filters with arbitrary stopband and equiripple passband responses is demonstrated. A new transformed frequency variable is used for iterative approximation with automatic bandwidth adjustment and prototype circuit realization.**

I. Introduction

A DESIGN method is presented using a new transformed frequency variable for narrow bandstop filters. With this method, the most general class of equiripple passband, narrow bandstop filters can be designed to meet arbitrary stopband requirements with symmetric or asymmetric frequency responses.

Symmetric frequency responses include conventional Chebyshev and elliptic function filters, and any other frequency-symmetric distribution of the stopband loss pole frequencies.

Asymmetric frequency responses include any other arbitrary distribution of the stopband loss pole frequencies. A simple example of an asymmetric response is shown in Figs. 1 and 2, which are the stopband and SWR responses of a five-resonator narrow bandstop filter. This filter has an asymmetric stopband with two bandstop resonators tuned to a lower frequency and three resonators tuned to a higher frequency. The resulting equiripple SWR response has two reflection minima in the lower passband, and three minima in the upper passband.

Both the approximation and realization steps of synthesis are performed in the new transformed variable, the use of which simplifies the iterative approximation of equiripple passband, general stopband responses, and, with care, achieves high numerical accuracy in the realization of the prototype element values [1]. Auxiliary transformed variables are included for automatic bandwidth adjustment in the approximation procedure [2]. The general steps taken in the narrow bandstop synthesis are similar to those used for the narrow bandpass filter [3], and will be used to develop a normalized highpass prototype.

Synthesis of the prototype filter will be performed using the transducer and characteristic functions, which in terms of the lossless, doubly-terminated two-port scattering parameters are $H = 1/S_{12}$ and $K = S_{11}/S_{12}$, respectively. The filter loss is $\alpha = 10\log_{10}(|H|^2)$ dB, and $|H|^2 = 1 + |K|^2$. H and K will be rational functions of a complex transformed frequency variable (z), chosen to map the filter passbands onto the entire imaginary axis in the z plane, and the stopband into the positive real axis.

Manuscript received March 29, 1991; revised August 3, 1991.
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IEEE Log Number 9103392.

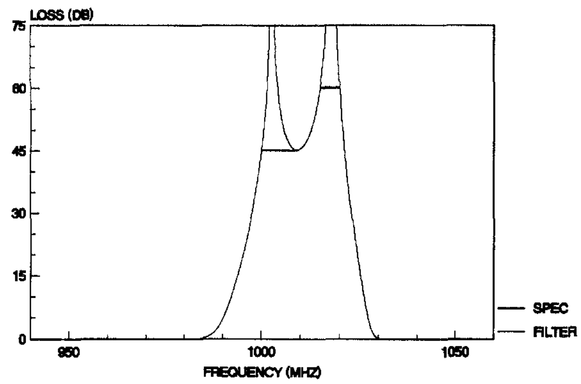

Fig. 1. Loss response, Chebyshev rational function.

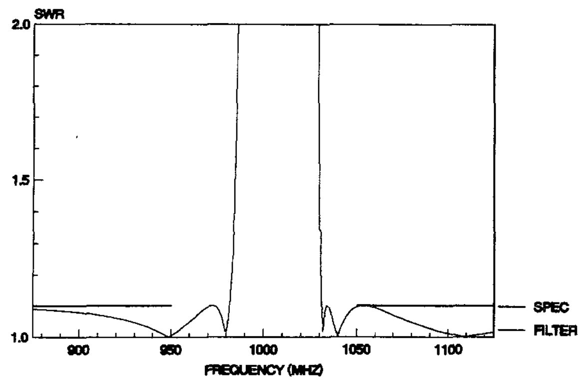

Fig. 2. SWR response, Chebyshev rational function.

II. Approximation

A. Filter Specifications

The narrow bandstop filter must pass signals below and above a pair of specified passband edge frequencies (p_1, p_2) with a specified maximum reflection, and it must reject signals between a pair of stopband edge frequencies (s_1, s_2) by specified values of minimum loss, where $p_1 < s_1 < s_2 < p_2$. The filter designer will obtain these values from the imposed requirements for the filter, modifying them to include margins for practical tolerances, tuning capability, environmental conditions, and any other "rule-of-thumb" based on experience.

0018-9480/91$01.00

For a specific filter design, the actual equiripple passband edge frequencies (f_1, f_2) may either be fixed or be determined automatically during the approximation step; a satisfactory design must meet the conditions $p_1 \leqslant f_1 < s_1$ and $s_2 < f_2 \leqslant p_2$. The quantity $f_2 - f_1$ will be referred to as the filter bandwidth.

Because there is no restriction on the symmetry of the frequency response, the definition of a "center" frequency f_0 becomes somewhat arbitrary. A very useful choice of f_0 is

$$f_0 = \frac{p_2 \cdot s_1 - p_1 \cdot s_2}{(p_2 - p_1) - (s_2 - s_1)}, \tag{1}$$

which divides the stopband width $(s_2 - s_1)$ and specified passband width $(p_2 - p_1)$ into similar proportions:

$$\frac{f_0 - s_1}{s_2 - f_0} = \frac{f_0 - p_1}{p_2 - f_0} = \frac{\Delta_1}{\Delta_2};$$

Δ_1 and Δ_2 are the fractional proportions split by f_0, i.e., $\Delta_1 + \Delta_2 = 1$.

B. Transformed Variables

The following transformation and its inverse

$$z^2 = \frac{f - f_1}{f_2 - f}, \qquad \mathrm{Re}(z) \geqslant 0, \qquad f = \frac{z^2 \cdot f_2 + f_1}{z^2 + 1} \tag{2}$$

accomplishes the desired mapping of the passbands $(f \leqslant f_1, f \geqslant f_2)$ and stopband $(s_1 \leqslant f \leqslant s_2)$ of a narrow bandstop filter.

By analogy to a narrow bandpass filter whose prototype is a normalized lowpass filter, the prototype for the narrow bandstop filter is a normalized highpass filter [4]. In the normalized frequency domain (ω) for the prototype the transformed variable and its inverse are

$$z^2 = \frac{1 + \omega}{1 - \omega}, \qquad \mathrm{Re}(z) \geqslant 0, \qquad \omega = \frac{z^2 - 1}{z^2 + 1}$$

where f_1 maps to $\omega = -1$, f_2 maps to $\omega = +1$, and the stopband frequencies map into $-1 < \omega < +1$.

Appropriate auxiliary transformed variables and their inverses for automatic bandwidth adjustment [2] are

$$x = \frac{f - s_1}{s_2 - f} \qquad f = \frac{x \cdot s_2 + s_1}{x + 1} \tag{3}$$

$$y = \frac{f_2 - f_1}{s_2 - s_1} - 1 \qquad f_2 - f_1 = (s_2 - s_1) \cdot (y + 1) \tag{4}$$

where x and y are the stopband and bandwidth transformed variables, respectively. Stopband loss poles will map onto $x_i > 0$, independently of the filter bandwidth, and the filter bandwidth, which is always greater than the stopband width, will map onto $y > 0$. The z-plane loss poles may be recovered from

$$z_i^2 = \frac{x_i + \Delta_1(x_i + 1) y}{1 + \Delta_2(x_i + 1) y},$$

which is well-conditioned, since all the values are positive and there are no subtractions.

A convenient way to relate f_1 and f_2 to the filter bandwidth is to require that f_0 will also divide the filter bandwidth into the same proportions as the other bandwidths. Then the transform of $f_0, z^2(f_0) = (f_0 - f_1)/(f_2 - f_0)$ is equal to Δ_1/Δ_2, independent of the bandwidth, and is given directly as

$$z_0^2 \equiv z^2(f_0) = \frac{s_1 - p_1}{p_2 - s_2}.$$

As a result, the actual passband edge frequencies (f_1, f_2) will vary so that in the limit that the filter bandwidth is equal to the stopband width $(y = 0)$, they will coincide with the stopband edge frequencies (s_1, s_2), and when the bandwidth is equal to the specified passband width, they will coincide with the specified passband edge frequencies (p_1, p_2).

C. Chebyshev Rational Function

Let z_i, $i = 1$ to n, refer to the stopband loss pole frequencies transformed by (2), where n is the filter degree. Since all z_i are positive numbers,

$$E + zF = \prod_{i=1}^{n} (z_i + z)^2$$

is a strictly Hurwitz polynomial whose even (E) and odd (zF) parts have alternating roots along the imaginary z-axis. Then

$$|K|^2 = \frac{k^2 E^2}{E^2 - z^2 F^2}$$

is a Chebyshev rational function [1]: in the passband $0 \leqslant |K|^2 \leqslant k^2$ is an optimum equiripple manner, and in the stopband the poles of $|K|^2$ are at $z = z_i$ where $E - zF = 0$.

The values of z_i may be obtained by transforming the loss poles of a known (e.g., elliptic function) response, or by iterative adjustment to meet a specified stopband response and passband ripple. The latter may be performed with a fixed bandwidth [1] resulting in a response with a free parameter such as excess stopband loss, or with adjustable bandwidth [2] using the auxiliary transformed variables (3) and (4).

III. Realization

A. Design Immittances

Prototype circuit elements may be extracted from a set of two-port immittance parameters obtained from the numerator polynomials of H and K [3]. Let $\mathrm{Den}(H) = \mathrm{Den}(K)$, $\mathrm{Num}(H) = H_R + jH_I$ and $\mathrm{Num}(K) = K_R + jK_I$, where H_R, H_I, K_R, and K_I are real polynomials in z^2. Choices in the form of these polynomials will be made as a matter of convenience and with no loss of generality for the prototype; other appropriate choices will lead to the same circuit except for different phase shifts at the input and output ports.

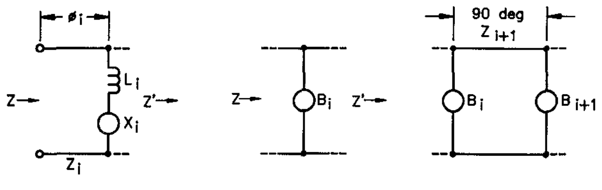

Fig. 3. Prototype circuit removals.

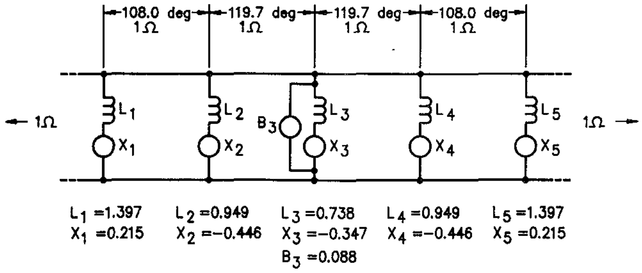

Fig. 4. Normalized highpass prototype.

Choosing $K_R = 0$, and finding the n quadratic factors $z^2 + r_i z + s_i$ of $(1+k^2)^{1/2} \cdot E + zF$, then

$$\text{Num}(K) = K_R + jK_I = jkE$$

$$\text{Num}(H) = H_R + jH_I = (1+k^2)^{1/2} \cdot \prod_{i=1}^{n} \left(z^2 - a_i - jb_i\right)$$

where $a_i = r_i^2/2 - s_i$ and $b_i = r_i(s_i - r_i^2/4)^{1/2}$. Assuming a prototype with unit terminations, the two-port open-circuit reactances and short-circuit susceptances are

$$x_{11} = x_{22} = -H_R/(H_I - K_I)$$
$$b_{11} = b_{22} = -H_R/(H_I + K_I) \tag{5}$$

respectively, resulting in a circuit which is electrically, but not necessarily physically, symmetric.

B. Prototype Development

Starting with an appropriate immittance function from (5), the normalized highpass prototype will be realized by repeated removal of sections consisting of a shunt bandstop resonator (producing a loss pole at $z(\omega_i) = z_i$), preceded by a phase shifter of characteristic impedance $Z_i = 1/Y_i$ and phase length ϕ_i, as shown in Fig. 3(a). The reactance of the resonator is $\omega L_i + X_i$, which is zero at the loss pole, so that $X_i = -\omega_i L_i$.

Let $Z = jU(z^2)/V(z^2)$ be the input impedance at the left of the section, and $Z' = jU'(z^2)/V'(z^2)$ be the input impedance of the remaining circuit. Specifying z_i and Z_i (normally 1 Ω) then ϕ_i and L_i may be determined by solving the polynomial equations

$$-V = -\cot\phi_i Y_i U + \left(z^2 - z_i^2\right) Y_i U' \tag{6}$$

$$Y_i U + \cot\phi_i V = Z_i \frac{z_i^2 + 1}{2L_i}\left(z^2+1\right) Y_i U' + \left(z^2 - z_i^2\right) V'. \tag{7}$$

These equations are of the general form

$$P(z^2) = aQ(z^2) + \left(z^2 - z_i^2\right) R(z^2)$$

where P and Q are knowns, and R and a are unknowns. Then (6) is solved with $P = -V$, $Q = Y_i U$, $R = Y_i U'$, and $a = -\cot\phi_i$, and (7) is solved with $P = Y_i U + \cot\phi_i V$, $Q = (z^2+1) Y_i U'$, $R = V'$, and $a = Z_i(z_i^2+1)/2L_i$.

Clearly, $a = P(z_i^2)/Q(z_i^2)$, and $R(z^2)$ can be solved by dividing $P(z^2) - aQ(z^2)$ by $z^2 - z_i^2$. However, this will lead to loss of numerical accuracy from terms which cancel each other [1], [3]. Instead, the following algorithm should be used, where terms in $R(z^2)$ are of the form $r_k \cdot z^{2k}$, $k = 0$ to m (and similarly for P and Q, which are of degree $m+1$):

$$r_k \cdot Q(z_i^2) = \left[\sum_{j=0}^{k} q_j \cdot z_i^{2j}\right] \cdot \left[\sum_{j=k+1}^{m+1} p_j \cdot z_i^{2(j-k-1)}\right] - \left[\sum_{j=0}^{k} p_j \cdot z_i^{2j}\right] \cdot \left[\sum_{j=k+1}^{m+1} q_j \cdot z_i^{2(j-k-1)}\right].$$

This equation was obtained by solving for the r_k recursively using both ascending and descending solutions, and eliminating the term a.

The phase shifter and resonator sections can be repeatedly removed until only a final phase shifter remains. Realizing the prototype from either end will result in the same circuit unless there is a residual mismatch at "infinite frequency" ($z^2 = -1$). The simplest ways to handle this are to remove a single shunt susceptance B_i at a resonator node as in Fig. 3(b) if no impedance transformation is required, otherwise remove a pair of susceptances B_i, B_{i+1} and an immittance inverter (90-degree line, non-unity Z_{i+1}) as shown in Fig. 3(c). The values of B_i, B_{i+1}, and Z_{i+1} are determined by examination of the element values as they are removed from opposite ends, and by the impedances and total phase shifts from each end at $z^2 = -1$. Removal of a shunt susceptance B_i from Z results in a new impedance Z' with $U' = U$ and $V' = V + B_i U$.

IV. Design Example

The five-resonator filter described above will be used to illustrate the synthesis. The filter specifications are: passband below $p_1 = 950$ MHz and above $p_2 = 1050$ MHz and 1.1 maximum SWR; stopband with 45 dB minimum from $s_1 = 1000$ to 1010 MHz and 60 dB minimum from 1015 to $s_2 = 1020$ MHz. The resulting center frequency from (1) is $f_0 = 1012.50$ MHz.

The loss response of the filter was optimized by adjusting the two loss pole frequencies and the filter bandwidth, so that the stopband loss exactly met the specifications. Figs. 1 and 2 are the resulting stopband and equiripple SWR responses, where $f_1 = 981.47$ MHz, $f_2 = 1031.11$ MHz, the loss pole frequencies are 1002.48 and 1017.95 MHz, and the frequency of the minimum in the stopband response is 1009.09 MHz.

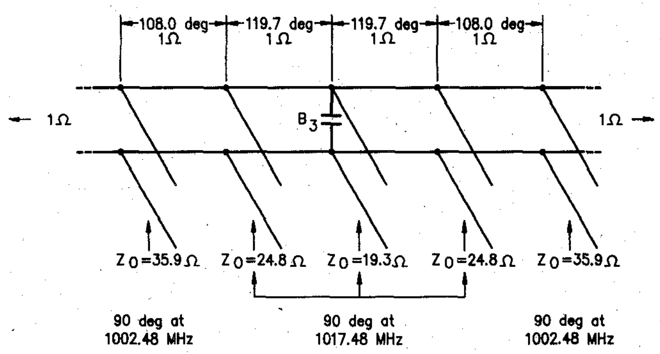

Fig. 5. Normalized transmission line bandstop prototype.

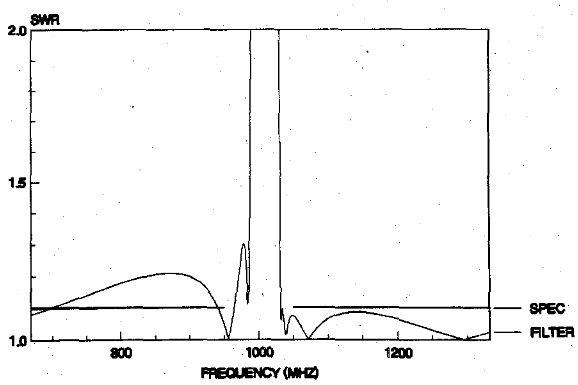

Fig. 6. SWR response, bandstop prototype.

A symmetric circuit, shown in Fig. 4, resulted from the removal of a lower frequency resonator from each end, with the three higher frequency resonators in the middle. The center resonator is shunted by a constant susceptance, which accounts for the mismatch in the response at infinite frequency.

Conversion of the highpass prototype into a normalized transmission line prototype is shown in Fig. 5, with the resulting SWR response shown in Fig. 6 (the stopband response is very close to that of the prototype). Depending on the filter structure chosen, this prototype must be converted again into a practical bandstop circuit. Further optimization (including tuning) will restore the SWR response.

References

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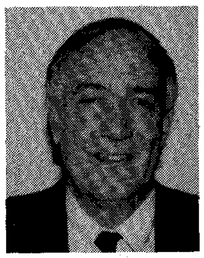

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From 1966 to 1968 he was with Rantec Corporation, Calabasas, CA, where he worked on ferrite phase shifters for phased array antennas and other microwave components. From 1969 to 1975 he was with Hughes Aircraft Company, El Segundo, CA, working on ferrite phase shifters, millimeter-wave solid-state components, and multiplexers for satellite transponders. He was with Wavecom, Northridge, CA, from 1975 to 1977, and Microwave Applications Group, Chatsworth, CA, from 1977 to 1981. He returned to Loral Microwave-Wavecom in 1981, and is now Vice President, Engineering. Dr. Bell has been responsible for a wide variety of filters, with emphasis on high power lumped-element filters in the HF-VHF-UHF bands, and coupled-resonator microwave filters including self-equalized and high-selectivity types.